EXAMPLE 5: Prove the following identity using Boolean algebra,

$\bar{A}\bar{B}C + \bar{A}BC + A\bar{B} = \bar{A}C + A\bar{B}$

Solution:

$$
\begin{aligned}
\text{L.H.S.} &= \bar{A}\bar{B}C + \bar{A}BC + A\bar{B} \\
&= \bar{A}C(\bar{B} + B) + A\bar{B} \\
&= \bar{A}C + A\bar{B} = \text{R.H.S.}
\end{aligned}
$$

EXAMPLE 6: Prove the following using Boolean algebra

$ABC + A\bar{B}C + \bar{A}BC + AB\bar{C} + A\bar{B}C + \bar{A}\bar{B}C = A + \bar{B}C$

Solution:

$ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}C = A + \bar{B}C$

$$
\begin{aligned}
\text{L.H.S.} &= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + \bar{A}\bar{B}C \\
&= AB\,(C + \bar{C}) + A\bar{B}\,(C + \bar{C}) + \bar{B}C(A + \bar{A}) \quad (\therefore C + C = 1, A + A = 1) \\
&= AB + A\bar{B} + \bar{B}C \\
&= A\,(B + \bar{B}) + \bar{B}C \\
&= A + \bar{B}C = \text{R.H.S.}
\end{aligned}
$$

EXAMPLE 7: Prove the following Boolean identity,

$ABC + \bar{A}\bar{B}C + \bar{A}BC + AB\bar{C} + \bar{A}\bar{B}\bar{C} = \bar{A}\bar{B} + B\,(A + C)$

Solution:

$ABC + \bar{A}\bar{B}C + \bar{A}BC + AB\bar{C} + \bar{A}\bar{B}\bar{C} = \bar{A}\bar{B} + B\,(\bar{A} + C)$

$$
\begin{aligned}
\text{L.H.S.} &= ABC + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}\bar{B}\bar{C} + AB\bar{C} \\
&= ABC + ABC + \bar{A}\bar{B}C + \bar{A}BC + AB\bar{C} + AB\bar{C} \\
&= ABC + \bar{A}BC + AB\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} \\
&= BC\,(A + \bar{A}) + AB(C + \bar{C}) + \bar{A}\bar{B}\,(C + C) \\
&= BC + AB + \bar{A}\bar{B} \\
&= \bar{A}\bar{B} + AB + BC \\
&= \bar{A}\bar{B} + B(A + C) = \text{R.H.S.}
\end{aligned}
$$

EXAMPLE 8: Simplify the following Boolean expressions.

(a) $AB + A\bar{B}C\,(\bar{B}\bar{C} + C) + \overline{AC}$

(b) $\bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + ABC$

Solution:

(a) $AB + A\bar{B}C\,(\bar{B}\bar{C} + C) + \overline{AC}$

$$
\begin{aligned}
&= AB + A\bar{B}\bar{B}C\bar{C} + A\bar{B}CC + \overline{AC} \\
&= AB + 0 + A\bar{B}C + \bar{A} + \bar{C} \\
&= \bar{C} + AB + \bar{A} + A\cdot\bar{B}C \\
&= \bar{C} + AB + (\bar{A} + A)\cdot(\bar{A} + \bar{B}C) \\
&= \bar{C} + AB + 1\cdot(\bar{A} + \bar{B}C) \\
&= \bar{A} + A\cdot B + \bar{C} + \bar{B}\cdot C \\
&= (\bar{A} + A)\cdot(\bar{A} + B) + (\bar{C} + \bar{B})\cdot(\bar{C} + C) \\
&= 1\cdot(\bar{A} + B) + (\bar{C} + \bar{B})\cdot 1 \\
&= \bar{A} + B + \bar{C} + \bar{B}
\end{aligned}
$$

$$= (B + \bar{B}) + \bar{A} + \bar{C}$$
$$= 1 + \overline{AC} = 1 \text{ Ans.}$$

(b) $\bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + ABC$
$$= \bar{A}B\bar{C} + \bar{A}BC + \bar{A}BC + A\bar{B}\bar{C} + ABC$$
$$= \bar{A}B(\bar{C} + C) + \bar{A}BC + A\bar{B}C + A\bar{B}\bar{C}$$
$$= \bar{A}B \cdot 1 + BC(\bar{A} + A) + A\bar{B}\bar{C}$$
$$= \bar{A} \cdot B + BC \cdot 1 + A\bar{B}\bar{C}$$
$$= \bar{A} \cdot B + BC + A\bar{B}\bar{C}$$
$$= B \cdot (\bar{A} + C) + A\bar{B}\bar{C} \text{ Ans.}$$

EXAMPLE 9: Simplify the following Boolean expressions,

(a) $AB\bar{C}\bar{D} + AB\bar{C}D + ABC\bar{D} + ABCD$

(b) $AB + AB\bar{C} + \bar{A}BC + ABC$

(c) $AB(\bar{A}B\bar{C} + A\bar{B}\bar{C} + \bar{A}BC)$

Solution:

(a) $AB\bar{C}\bar{D} + AB\bar{C}D + ABC\bar{D} + ABCD$
$$= AB\bar{C}(\bar{D} + D) + ABC(\bar{D} + D)$$
$$= AB\bar{C} \cdot 1 + ABC \cdot 1$$
$$= AB\bar{C} + ABC$$
$$= AB(\bar{C} + C) = AB \cdot 1 = AB \text{ Ans.}$$

(b) $AB + A\bar{B}\bar{C} + ABC + ABC$
$$= AB + AB\bar{C} + \bar{A}BC + ABC + ABC$$
$$= AB + AB\bar{C} + \bar{A}BC + ABC + ABC$$
$$= AB + AB(\bar{C} + C) + BC(\bar{A} + A)$$
$$= AB + AB \cdot 1 + BC \cdot 1$$
$$= AB + AB + BC = AB + BC$$
$$= B \cdot (A + C) \text{ Ans.}$$

(c) $AB(\bar{A}B\bar{C} + A\bar{B}\bar{C} + \bar{A}BC)$
$$= AB\bar{A}B\bar{C} + ABA\bar{B}\bar{C} + AB\bar{A}BC$$
$$= A\bar{A} \cdot B\bar{C} + A\bar{B} \cdot \bar{B}\bar{C} + A\bar{A} \cdot BC$$
$$= 0 \cdot B\bar{C} + A \cdot 0 \cdot \bar{C} + 0 \cdot BC$$
$$= 0 + 0 + 0 = 0 \text{ Ans.}$$

EXAMPLE 10: Simplify the following Boolean expressions,

(a) $(\overline{A + B}) \cdot (\overline{\bar{A} + \bar{B}})$

(b) $ABC + \bar{A}B + AB\bar{C}$

(c) $C(A\bar{B} + AB) + BC$

Solution:

(a) $(\overline{A + B}) \cdot (\overline{\bar{A} + \bar{B}})$
$$= (\bar{A} \cdot \bar{B}) \cdot (\overline{\overline{A \cdot B}})$$
$$= (\bar{A} \cdot \bar{B}) \cdot (A \cdot B)$$
$$= A\bar{A} \cdot B\bar{B}$$
$$= 0 \cdot 0 = 0 \text{ Ans.}$$

Solution:

$$\begin{aligned}
\text{L.H.S.} &= (A + B)(\bar{A} + C) \\
&= A\bar{A} + AC + \bar{A}B + BC \\
&= 0 + AC + \bar{A}B + BC(A + \bar{A}) \\
&= 0 + AC + \bar{A}B + ABC + \bar{A}BC \\
&= AC + ABC + \bar{A}B + \bar{A}BC \\
&= AC(1 + B) + \bar{A}B(1 + C) \\
&= AC \cdot 1 + \bar{A}B \cdot 1 \\
&= AC + \bar{A}B = \text{R.H.S.}
\end{aligned}$$

EXAMPLE 16: Realise or implement the following Boolean expressions using basic gates.

(a) $Y = A \cdot B + C \cdot D + E \cdot F$

(b) $Y = (A + B) \cdot (C + D) \cdot (E + F)$

(c) $Y = (A + B) \cdot C + AB = AB + BC + CA.$

Solution:

(a) $Y = A \cdot B + C \cdot D + E \cdot F$

The implementation of the above Boolean expression is by the logic diagram of Fig. 2.13. Three separate AND gates are used to get product terms $A \cdot B$, $C \cdot D$ and $E \cdot F$ from inputs A, B, C, D, E, F. Then these three products terms which are outputs of AND gates are fed to input of an OR gate which gives final output $Y = A \cdot B + C \cdot D + E \cdot F$.

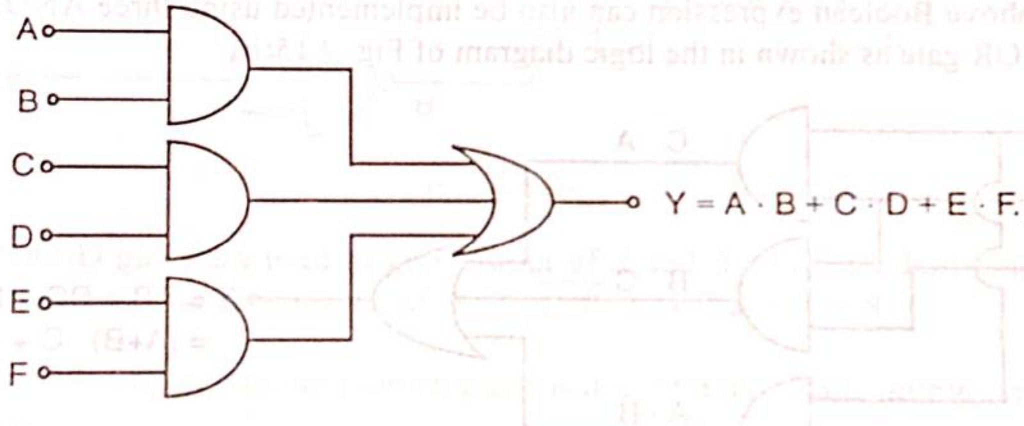


Fig. 2.13

(b) $Y = (A + B) \cdot (C + D) \cdot (E + F)$

The above Boolean expression can be realised by the logic diagram of Fig. 2.14.

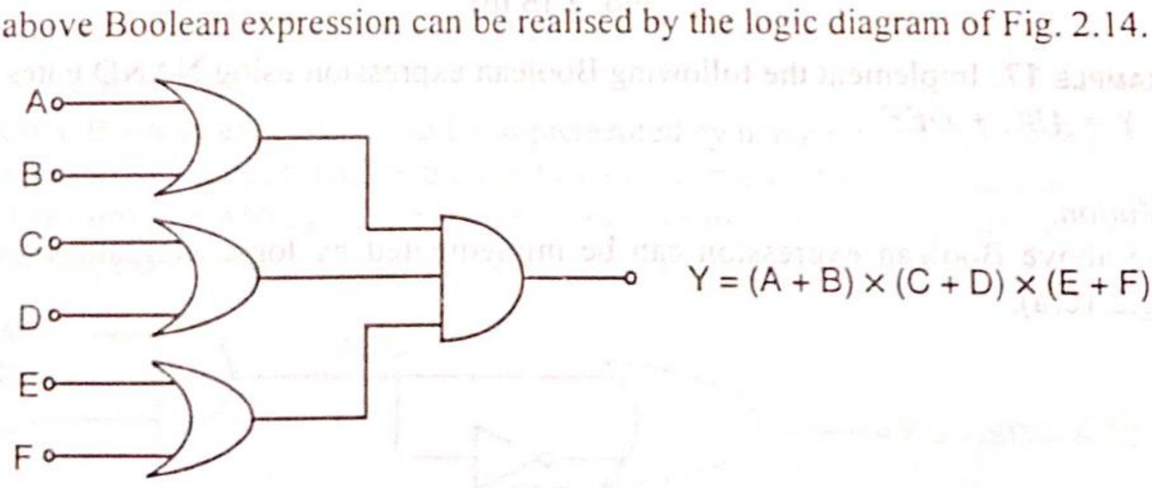


Fig. 2.14

Three OR gates are used to give the three sum terms of $(A + B)$,$(C + D)$ and $(E + F)$ which are fed as input to an AND gate to give the final output $Y = (A + B)(C + D)(E + F)$ as shown above.

(c) $Y = (A + B) \cdot C + AB = AB + BC + CA.$

The above Boolean expression can be implemented by the logic diagram Fig. 2.15(a) shown below.

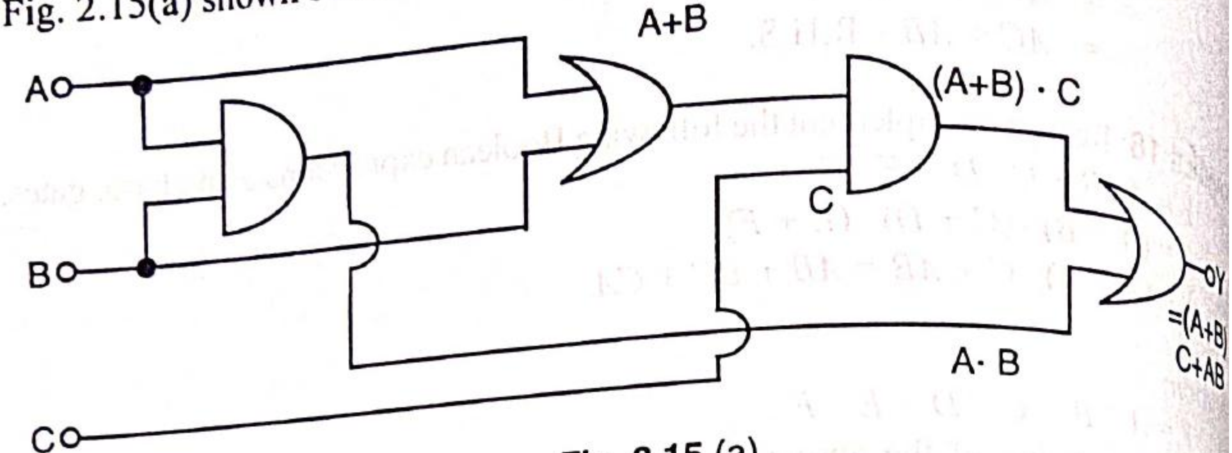


Fig. 2.15 (a)

An AND gate is used to give output of $A \cdot B$ and an OR gate to give output $A + B$. The output $A + B$ of OR gate is fed along with input C to the second AND gate which gives output $(A + B)\,C$. This output and output of first AND gate are fed to an OR gate, which gives final output $Y = (A + B)\,C + AB$.

The above Boolean expression can also be implemented using three AND gates and one OR gate as shown in the logic diagram of Fig. 2.15(b)

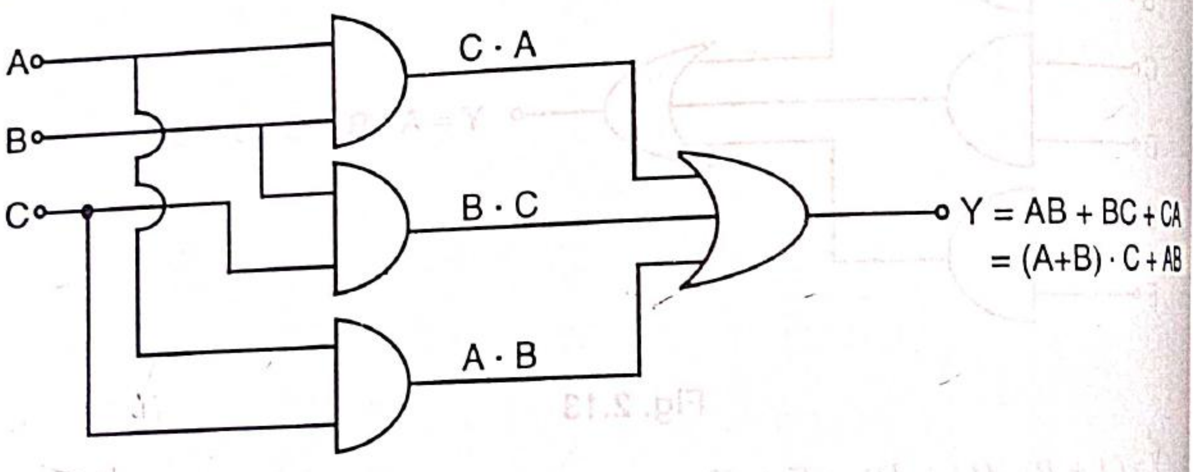


Fig. 2.15 (b)

EXAMPLE 17: Implement the following Boolean expression using NAND gates only

$Y = ABC + DEF$

Solution:

The above Boolean expression can be implemented by logic diagram shown in Fig.2.16(a).

(b) $Y = A + \bar{B}C$

The above Boolean expression can be implemented by logic diagram shown below in Fig. 2.18

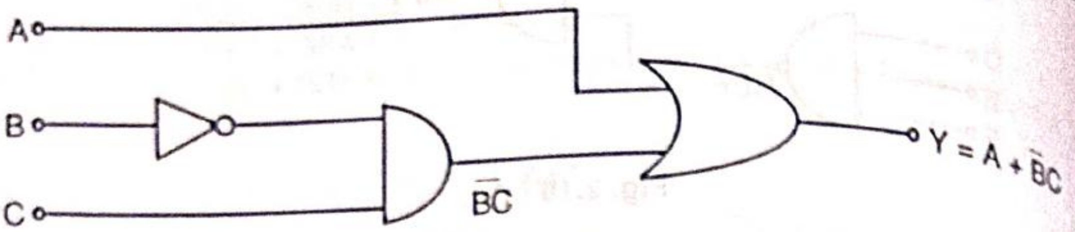


Fig. 2.18

Input B after passing through the inverter is fed alongwith input C to AND gate which gives output $\bar{B}C$. The output of AND gate and Input A are fed to an OR gate to give the final output $A + \bar{B} \cdot C$.

EXAMPLE 20: Implement the following Boolean expression using NAND gates only

(a) $Y = A + \bar{B}C + AC$

(b) $Y = \overline{(A + \bar{B}) \cdot (\bar{A} + C)}$

Solution:

(a) Here $Y = A + \bar{B}C + AC$

$$\therefore Y = \overline{\overline{A + \bar{B}C + AC}}$$

$$= \overline{\bar{A} \cdot \overline{\bar{B}C} \cdot \overline{AC}} \quad \text{(using De Morgan's Theorem)}$$

$$\therefore Y = Y = A + \bar{B}C + AC$$

Thus it can be easily realised using NAND gates as shown below in Fig. 2.19(a).

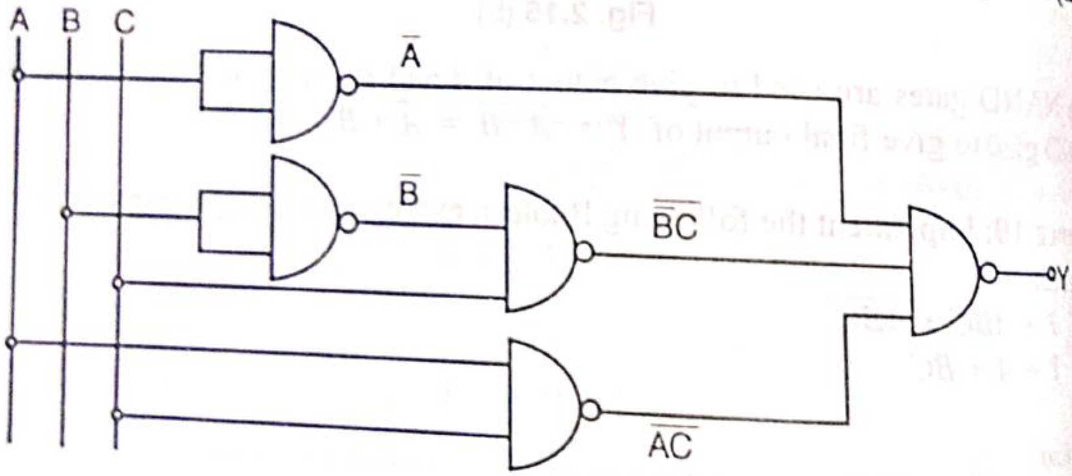


Fig. 2.19 (a)

(b) $Y = \overline{(A + \bar{B}) \cdot (\bar{A} + C)}$

$$= \overline{(A + \bar{B})} + \overline{(\bar{A} + C)} \quad \text{(using De Morgan's Theorem)}$$

$$= \bar{A} \cdot \bar{\bar{B}}) + (\bar{\bar{A}} \cdot \bar{C}) \quad \text{(using De Morgan's Theorem)}$$

$$= \bar{A} \cdot B + A\bar{C}$$

$$\therefore Y \quad \overline{\overline{\bar{A} \cdot B + A\bar{C}}}$$

$$= \overline{\overline{\bar{A}B} \cdot \overline{A\bar{C}}}$$

$$\therefore Y = \overline{\overline{\bar{A}B} \cdot \overline{A\bar{C}}}$$

Thus it can be easily realised using NAND gates as shown in Fig. 2.19(b).

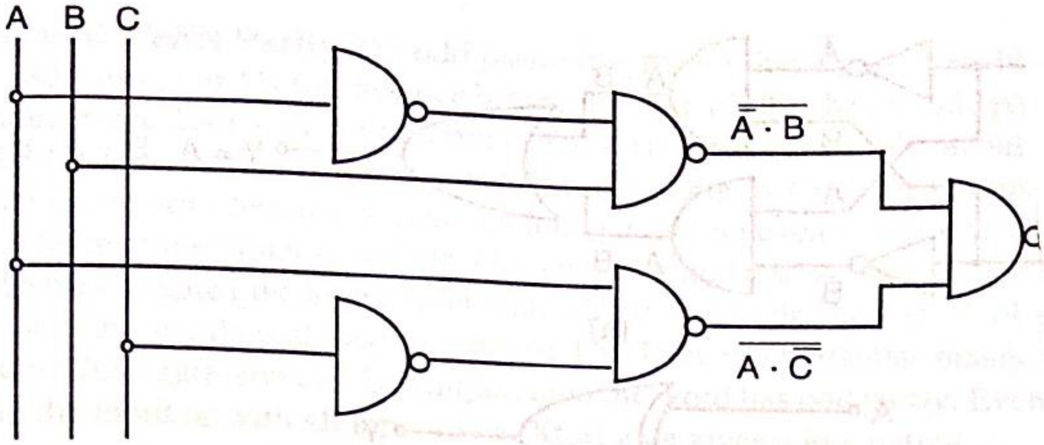


Fig. 2.19 (b)

EXAMPLE 21: Implement the following Boolean expression using basic gates.

$$Y = \overline{A}\overline{C} + \overline{B}\overline{C} + BC + AC$$

Solution:

$$Y = \overline{A}\overline{C} + \overline{B}\overline{C} + BC + AC$$

The above Boolean expression can be realised by logic diagram of Fig. 2.20, shown below.

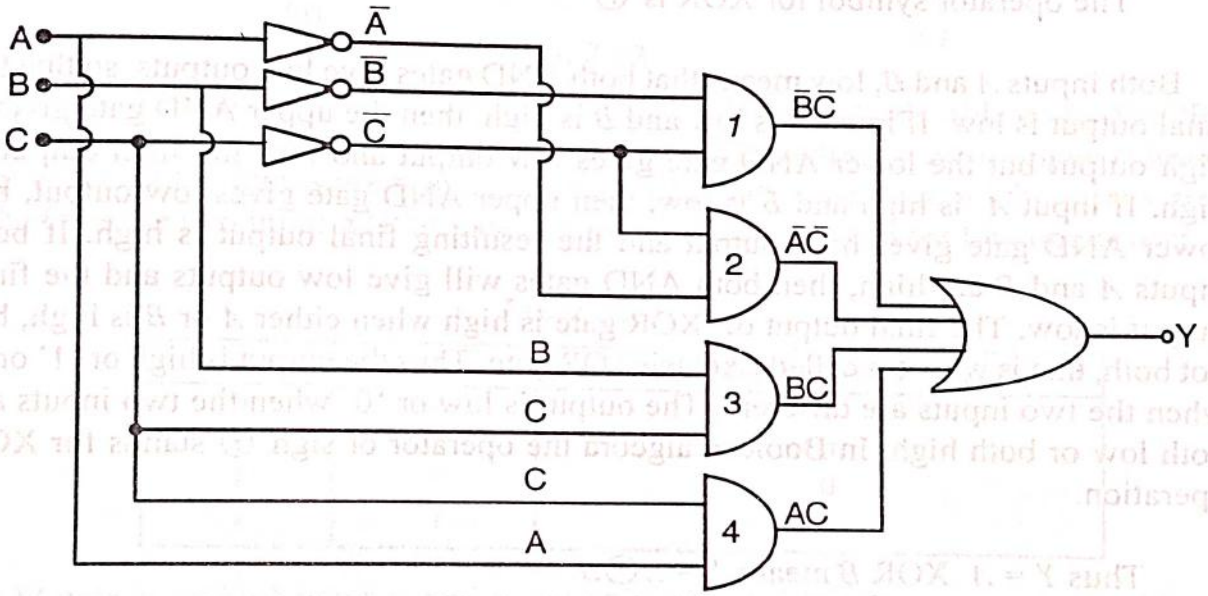
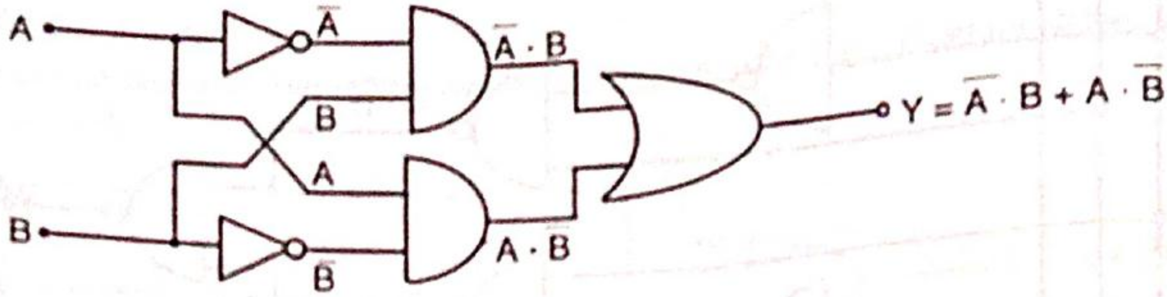


Fig. 2.20

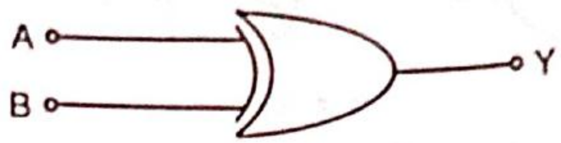
Three inverters (NOT gates) are used to give outputs of $\overline{A}$, $\overline{B}$ and $\overline{C}$. Then AND gates 1 and 2 are used to give output of $\overline{B}\overline{C}$ and $\overline{A}\overline{C}$ respectively. AND gates 3 and 4 are used to give outputs of BC and AC. Finally an OR gate is used to give a final output of $Y = \overline{B}\overline{C} + \overline{A}\overline{C} + BC + AC$ as shown above in the logic diagram.

2.9 EXCLUSIVE OR GATE

Exclusive OR gate is abbreviated as XOR gate. Figure 2.21 (a) shows the logic diagram and Fig. 2.21(b) shows its standard symbol.



(a)



(b) Standard Symbol of XOR gate

Fig. 2.21

Two inverters give outputs of $\bar{A}$ and $\bar{B}$. Then two AND gates are used. The upper AND gate gives the product $\bar{A}B$ and lower AND gate gives the product $A \cdot \bar{B}$ as shown in Fig. 2.21(a). Finally $\bar{A}B$ and $A\bar{B}$ are given as input to an OR gate and the resulting output is

$$Y = \bar{A}B + A \,.\, \bar{B}$$

The operator symbol for XOR is $\oplus$

Both inputs A and B, low means that both AND gates give low outputs, so that the final output is low. If input A is low and B is high, then the upper AND gate gives a high output but the lower AND gate gives low output and thus the final output is high. If input A is high and B is low, then upper AND gate gives low output, but lower AND gate gives high output and the resulting final output is high. If both inputs A and B are high, then both AND gates will give low outputs and the final output is low. The final output of XOR gate is high when either A or B is high, but not both, that is why it is called Exclusive OR gate. Thus the output is high or '1' only when the two inputs are different. The output is low or '0' when the two inputs are both low or both high. In Boolean algebra the operator or sign $\oplus$ stands for XOR operation.

Thus $Y = A$ XOR B means $Y = A \oplus B$.

The truth table of exclusive OR gate i.e., XOR gate is given below in Table 2.10.

Table 2.10

A	B	$Y = A \oplus B = \bar{A}B + A \cdot \bar{B}$
0	0	0
0	1	1
1	0	1
1	1	0

ODD and EVEN Parity By odd parity it is meant that a particular binary number has odd number of 1's for instance binary number 101001 has an odd parity becase it contains three 1's i.e. an odd number of 1's. By even parity it is meant that the particular binary number has an even number of 1's. For instance binary number 101101 has an even parity because it contains four 1's i.e., an even number of 1's. Exclusive OR gates are most suitable and ideal for checking and testing the parity of a word. The Exclusive OR gate give high output with odd parity words. As such XOR gates recognise only the words with odd number of 1's. Thus if a particular binary word as an input to XOR gate gives a high output, then the word has odd partiy. Even number of 1's in the input or with all inputs low, XOR gate gives a low output.

2.10 EXCLUSIVE NOR GATE

Exclusive NOR is abbreviated as XNOR gate. Figure 2.22(a) shows the logic diagram of XNOR gate and Fig. 2.22(b) shows its standard symbol. The logical equivalence of Exclusive NOR gate is XOR gate followed by NOT gate. A two input XNOR gate is shown in the figure below.

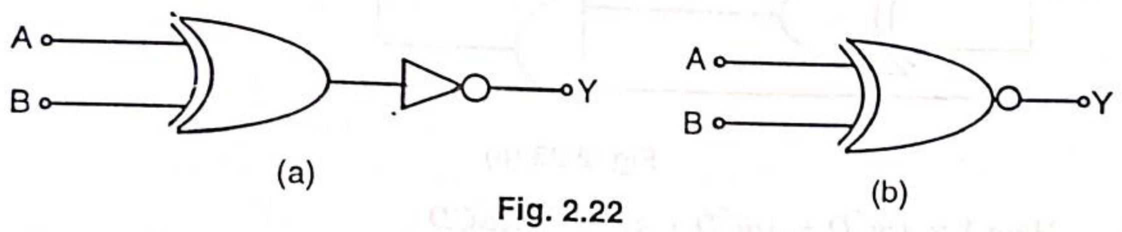


Fig. 2.22

Due to the NOT gate on the output side in Fig. 2.21(a), the truth table of an XNOR gate is the complement of truth table of XOR gate. The output of XNOR gate is high only when all the inputs are same i.e., either all inputs are low or all the inputs are high. Thus the two input XNOR gate is most suitable and ideal for bit comparison.

Table 2.11

A	B	$Y = A \odot B = \bar{A} \cdot \bar{B} + A \cdot B$
0	0	1
0	1	0
1	0	0
1	1	1

XNOR gate gives high input or recognises only when the two inputs are identical i.e., either both inputs are low or both inputs are high. Due to NOT gate (Inverter) on the output side of Fig. 2.21(a), the XNOR gate performs the complementary operation of XOR gate i.e., gives an output $Y = \overline{A \oplus B}$, which is represented symbolically by $A \odot B$. Thus operator $\odot$ stands for XNOR operation. Instead of recognising the odd parity words, the Exclusive NOR gate recognises the even parity words. In case of two input XNOR gate, both inputs high or both inputs low, give a high output as is shown in the first and last entry of Truth Table 2.11. Output of XNOR gate is

$$
\begin{aligned}
Y &= \overline{A \oplus B} = \overline{A\bar{B} + \bar{A}B} = \overline{A\bar{B}} \cdot \overline{\bar{A}B} \\
&= (\bar{A} + \bar{\bar{B}}) \cdot (\bar{\bar{A}} + \bar{B}) = (\bar{A} + B) \cdot (A + \bar{B}) = A\bar{A} + \bar{A}\bar{B} + AB + B\bar{B} \\
Y &= A \odot B = \bar{A}\bar{B} + AB
\end{aligned}
$$

EXAMPLE 22: (a) Implement the following Boolean expression using 4 logic gates only. $Y = AD + BC\bar{D} + \bar{B}CD$.

(b) Implement the given Boolean expression using 3 logic gates only $Y = \bar{A}\bar{B}\bar{C}\bar{D} + AB\bar{C}\bar{D} + ABCD + \bar{A}\bar{B}CD$

Solution:

(a) $Y = AD + BC\bar{D} + \bar{B}CD$

$= AD + C(B\bar{D} + \bar{B}D)$

$Y = A \cdot D + C \cdot (B \oplus D)$

Thus this Boolean expression can be realised by using 2 AND gates, 1 XOR gate and one OR gate as shown in Fig. 2.23 (a).

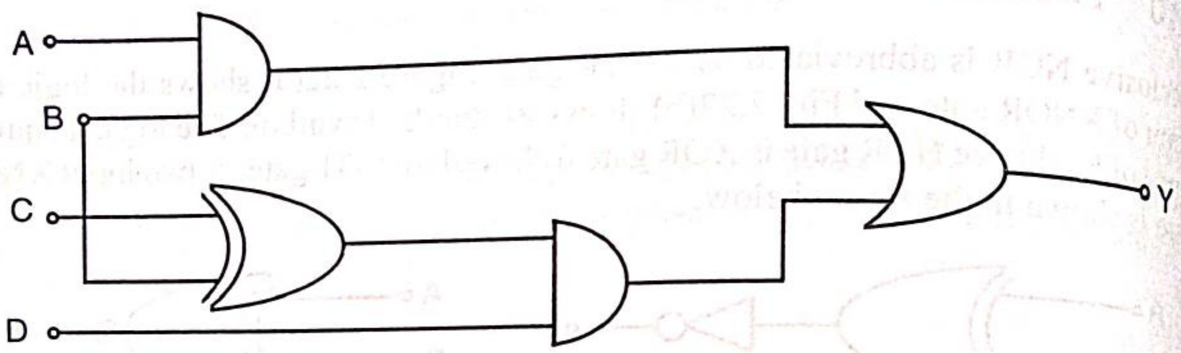


Fig. 2.23 (a)

(b) Here $Y = \bar{A}\bar{B}\bar{C}\bar{D} + AB\bar{C}\bar{D} + ABCD + \bar{A}\bar{B}CD$

$= \bar{C}\bar{D}(\bar{A}\bar{B} + AB) + CD(AB + \bar{A}\bar{B})$

$= \bar{C}\bar{D}(\bar{A}\bar{B} + AB) + CD(\bar{A}\bar{B} + AB)$

$= (\bar{A}\bar{B} + AB)(\bar{C}\bar{D} + CD)$

$= (A \odot B)(C \odot D)$

Which can be easily implemented using two XNOR gates and one AND gate as shown in Fig. 2.23 (b).

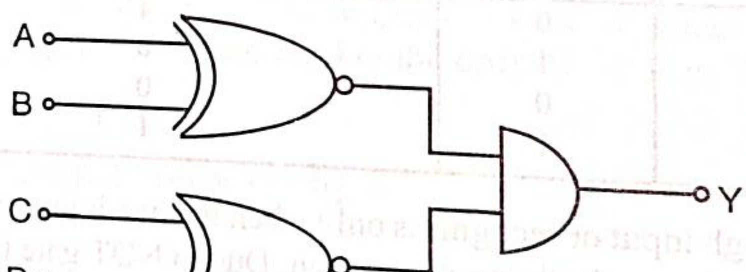


Fig. 2.23 (b)

EXAMPLE 23: Prove the following Boolean identities.

(i) $\bar{A} \odot B = A \oplus B$

(ii) $A \oplus B \oplus A \cdot B = A + B$

[AMIE Summer 1993]

Solution:

(i) $\bar{A} \odot B = A \oplus B$

(d) NOR

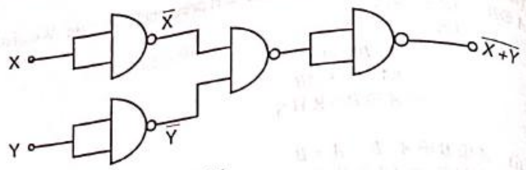


Fig. 2.27

(e) XOR

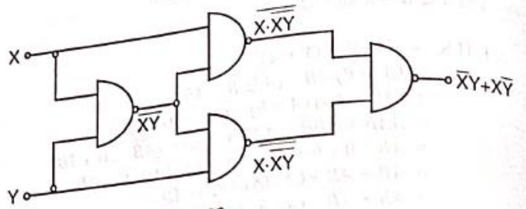


Fig. 2.28

(f) XNOR

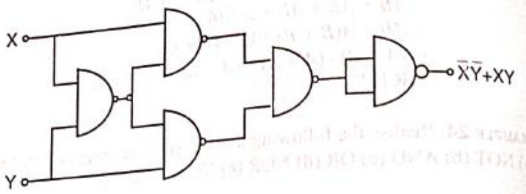


Fig. 2.29

EXAMPLE 25: Realise the following logic operations using only NOR gates: (a) NOT (b) AND (c) OR (d) NAND (e) XOR (f) XNOR.

Solution:

(a) NOT

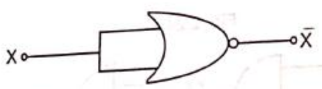


Fig. 2.30

(b) AND

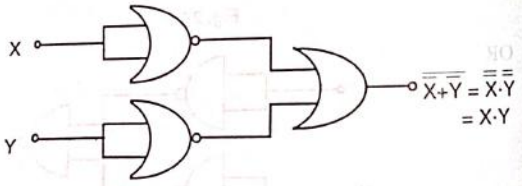


Fig. 2.31

(c) OR

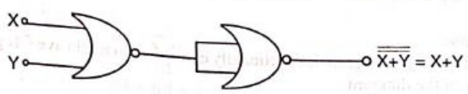


Fig. 2.32

(d) NAND

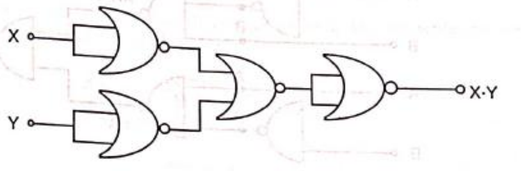


Fig. 2.33

(e) XOR

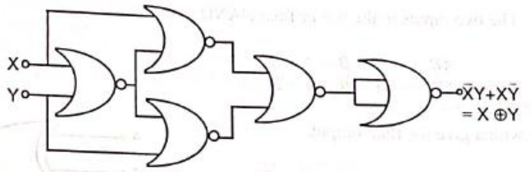


Fig. 2.34

(f) XNOR

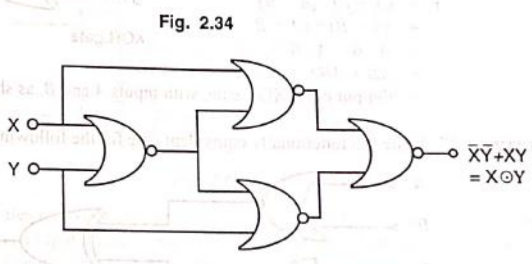


Fig. 2.35

EXAMPLE 26: Write the functionally equivalent gate for the following figure.
[AMIE Summer 1994]

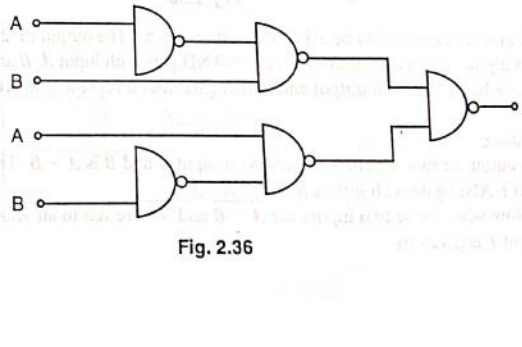


Fig. 2.36